

Niveau lycée

EXERCICES DE PERFECTIONNEMENT EN CALCUL

2^e édition

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SIMPLIFICATION D'EXPRESSIONS

EXERCICES

Pour tous les exercices de ce chapitre, simplifier au maximum l'expression donnée.

Pour s'échauffer...

$$A = \frac{1}{7} + \frac{2}{5}$$

$$B = \frac{3}{7} \times \frac{14}{9}$$

$$C = \sqrt{75}$$

$$D = \sqrt{30} \times \sqrt{15}$$

$$E = 5^4 \times 5^3$$

$$F = \frac{5^3}{5^2}$$

$$G = 2 + \frac{1}{4}$$

$$H = \frac{8}{7} - \frac{9}{2}$$

$$I = \sqrt{\frac{9}{16}}$$

$$J = 2\sqrt{\frac{25}{16}}$$

$$K = (2^4)^6$$

$$L = 49^5$$

Exercise 1.1

$$A = \frac{\frac{3}{8}}{\frac{6}{12}}$$

$$B = \frac{\frac{3}{8}}{\frac{6}{12}}$$

$$C = \frac{\frac{3}{8}}{\frac{6}{12}}$$

$$D = (\sqrt{2} - 3)(\sqrt{2} + 3)$$

$$E = \sqrt{3^2 + 4^2}$$

$$F = \sqrt{3^2} + \sqrt{4^2}$$

$$G = \frac{3}{7} + \frac{3}{5}$$

$$H = (3 + \sqrt{2})^2$$

$$I = (3\sqrt{2})^2$$

$$J = -\frac{5}{18} - \frac{7}{6} + \frac{3}{9} - 2$$

$$K = 3 \times 3^n$$

$$L = \frac{9 \times 10^3}{1,8 \times 10^5}$$

$$M = 2\sqrt{3} - 7\sqrt{3} + 2\sqrt{27}$$

$$N = \frac{1}{\frac{1}{2} + \frac{1}{3}}$$

Exercice 1.2

$$a = \frac{0,25}{2} \times 6$$

$$b = \frac{5 \times 4^n}{3 \times 2^n} \text{ où } n \in \mathbb{N}$$

$$c = \frac{0,8^2 \times 4^4 \times 9^3}{0,6^2 \times 12^5}$$

$$d = \left(\frac{2}{3}\right)^2 + 5 \times \frac{4}{3}$$

$$e = \sqrt{3^5 \times 15 \times 5^3}$$

$$f = \frac{\frac{14}{15}}{\frac{7}{12}}$$

$$g = \frac{15 \times 5^{n+1}}{15^{n+2}} \text{ où } n \in \mathbb{N}$$

$$h = \frac{7}{3} + \frac{5}{7}$$

$$i = (2 - \sqrt{3})^2$$

$$j = \sqrt{2^3 \times 49} \times \sqrt{\frac{2^7 \times 9}{4900}}$$

$$k = \frac{8^2 \times 5^5 \times 7}{2^8 \times 5^4 \times 7^2}$$

$$l = \frac{\frac{3}{2} - \frac{5}{6}}{\frac{1}{4} + \frac{1}{3}}$$

$$m = \sqrt{98} + \sqrt{32} - \sqrt{8}$$

$$n = \frac{4 - \frac{2}{3}}{4}$$

Exercice 1.3

Dans tout cet exercice, $n, p \in \mathbb{N}$.

$$A = \frac{7}{3} - \frac{\frac{8}{3}}{\frac{5}{16}}$$

$$B = \frac{5 \times 10^3 \times 2}{0,25 \times 10^{-4}}$$

$$C = \frac{4 \times 2^{n+2}}{2^n}$$

$$D = \frac{12}{5} \begin{pmatrix} \frac{1}{2} - \frac{1}{3} \\ \frac{1}{3} - \frac{1}{4} \end{pmatrix} \begin{pmatrix} \frac{1}{7} - \frac{1}{5} \\ \frac{5}{7} - \frac{7}{5} \end{pmatrix}$$

$$E = (\sqrt{3} + \sqrt{5})^2 - 2\sqrt{15}$$

$$F = 7^{n+1} - 3 \times 7^n$$

$$G = 3 \left(2 - \frac{1}{3} \right)^2 - \frac{5}{2}$$

$$H = 2\pi - \frac{\pi - 7}{5}$$

$$I = \begin{pmatrix} \frac{1}{2} - \frac{3}{4} \\ \frac{1}{3} - \frac{4}{5} \end{pmatrix} \begin{pmatrix} \frac{4}{15} - \frac{1}{3} \\ \frac{1}{6} + \frac{1}{4} \end{pmatrix}$$

$$J = \frac{(\sqrt{3})^{2p}}{3^p}$$

$$K = \frac{\frac{1}{3} - 3}{\left(\frac{2}{3} - 2\right)^2}$$

$$L = \frac{3}{2} (\sqrt{2} - \sqrt{3})^2 + 3\sqrt{6}$$

$$M = 2^{2n} + 3 \times 4^n$$

$$N = \frac{\frac{4}{5} + \frac{5}{4}}{3 \times \frac{7}{2} - 2}$$

Exercice 1.4

Dans tout cet exercice, $n \in \mathbb{N}$.

$$A = 2 \times 10^7 \times 5 \times \frac{(10^{-3})^5}{17+3}$$

$$B = 5 + \frac{\pi - 3}{7}$$

$$C = \frac{3\sqrt{28} - 4}{-2 + 3\sqrt{7}}$$

$$D = 5(4\sqrt{2})^2$$

$$E = 6 \times 3^n + 2 \times 3^{n+2}$$

$$F = \frac{2^8 - 2^{10}}{2^8 + 2^{10}}$$

$$G = 5 + (4\sqrt{2})^2$$

$$H = \frac{-21}{\frac{3}{-7} + \frac{3}{5}}$$

$$I = 2\sqrt{27} - \sqrt{15} \times 3\sqrt{45}$$

$$J = \frac{49}{24} \times \frac{25}{14} \times \frac{6}{5}$$

$$K = (\sqrt{3} - \sqrt{7})^2 + (\sqrt{7} + \sqrt{3})^2$$

$$L = \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} + \sqrt{3}}$$

$$M = \frac{2}{3} + \frac{3}{2} - \frac{1}{5}$$

$$N = \frac{\sqrt{3} \times 7\sqrt{12}}{2\sqrt{27}}$$

$$O = 3^n \left(\left(\frac{2}{3} \right)^n - \left(\frac{2}{3} \right)^{n+1} \right)$$

$$P = \frac{\frac{7}{5} - \frac{5}{7}}{4}$$

$$Q = \frac{10}{33} \times \frac{55}{4} \times \frac{1}{25}$$

Exercice 1.5

Dans tout cet exercice, $n \in \mathbb{N}$ et $a, b \in \mathbb{R}^*$.

$$A = -\frac{2n+3}{2^n} + \frac{4(n+1)+1}{2^{n+1}}$$

$$B = \frac{\frac{49}{24} \times \frac{25}{14} \times \frac{42}{5}}{\frac{2}{2}}$$

$$C = \frac{(9b^n)^2}{3b^2}$$

$$D = \frac{4 \times 2^{n+2}}{4^n}$$

$$E = \frac{2 - 4\sqrt{5}}{3 + 2\sqrt{5}}$$

$$F = \frac{\left((1 + \sqrt{5})^2 - (1 - \sqrt{5})^2 \right)^{2n}}{\left((2\sqrt{5})^2 \right)^n}$$

$$G = 3(\sqrt{3})^3 - 2\sqrt{75} + 5\sqrt{3^5}$$

$$H = \frac{7^{n+1} - 3 \times 7^n}{2 \times 7^n}$$

$$I = \frac{3 \left(\frac{5}{7} - \frac{1}{3} \right) \left(\frac{1}{4} + \frac{5}{2} \right)}{3 - \frac{7}{6}}$$

$$J = \frac{a^{2n} + 2a^n}{4a^n}$$

$$K = 1 - \frac{\frac{2}{3} \left(1 - \frac{7}{2} \right)}{8 - \frac{5}{3}}$$

$$L = \frac{3 - \frac{2}{3} \times \frac{5-2}{8-2}}{1 - 2 \times \frac{9}{8}}$$

$$M = \left(\frac{2 - \sqrt{3}}{\sqrt{3} - 1} \right)^2$$

$$N = \frac{\left(3 - \frac{2}{3} \right) \times \frac{5-2}{8-2}}{(1-2) \times \frac{9}{8}}$$

Exercice 1.6

Dans tout cet exercice, $n, p \in \mathbb{N}$.

$$A = (1 - \sqrt{5})^4 - (2\sqrt{5})^3$$

$$B = 2 - \frac{2 \times \left(\frac{7}{4} - \frac{5}{3}\right)}{\left(\frac{1}{5} + \frac{3}{2}\right) \left(\frac{5}{2} - \frac{5}{3}\right)}$$

$$C = \sqrt{6} + \frac{\sqrt{2} + \sqrt{3}}{2\sqrt{2} - 5\sqrt{3}}$$

$$D = (4 - 3\sqrt{2})^2 + 3(\sqrt{2} - 1)^2$$

$$E = \frac{\frac{1}{2} - \frac{1}{4} \times \left(3 + \frac{1}{3}\right)}{\left(\frac{1}{2} - \frac{1}{4}\right) \times 3 - \frac{1}{3}}$$

$$F = \frac{5 + 7\sqrt{2}}{\sqrt{2} - \sqrt{3}} + \frac{7 - 5\sqrt{3}}{\sqrt{2} + \sqrt{3}}$$

$$G = (\sqrt{2})^{2n} + \frac{6^{n+1}}{5 \times 3^n}$$

$$H = \frac{\frac{1}{4} \times \left(2 - \frac{4}{5}\right) - \frac{5}{16}}{\frac{8}{15} \times 3 + \frac{6}{5}}$$

$$I = \frac{6 \frac{5 \times \frac{3}{8} + \frac{7}{12} - 1}{7}}{-4 \times \frac{9}{15} + 3}$$

$$J = \frac{6 \times 9^n + 2 \times 3^{2n+2}}{6^3}$$

$$K = \frac{18^n + 2^n \times 3^{2n+2}}{6^{n-2}}$$

$$L = \frac{4}{7} (\sqrt{14})^{2p+2}$$

$$M = \frac{10}{2\sqrt{5} - 5} - 2 \frac{7 - 3\sqrt{5}}{\sqrt{20} + 5}$$

$$N = 4 \left(2 - \frac{\sqrt{3}}{2}\right)^3$$

CORRECTION DES EXERCICES

Echauffement

$$A = \frac{1}{7} + \frac{2}{5} \stackrel{F_0}{=} \frac{1 \times 5}{7 \times 5} + \frac{2 \times 7}{5 \times 7} = \frac{5}{35} + \frac{14}{35} \stackrel{F_1}{=} \frac{5+14}{35} = \boxed{\frac{19}{35}}$$

$$B = \frac{3}{7} \times \frac{14}{9} \stackrel{F_2}{=} \frac{3 \times 14}{7 \times 9} = \frac{3 \times 2 \times 7}{7 \times 3 \times 3} \stackrel{F_0}{=} \boxed{\frac{2}{3}}$$

$$C = \sqrt{75} = \sqrt{25 \times 3} \stackrel{R_1}{=} \sqrt{25} \times \sqrt{3} = \sqrt{5^2} \times \sqrt{3} \stackrel{R_0}{=} \boxed{5\sqrt{3}}$$

$$D = \sqrt{30} \times \sqrt{15} = \sqrt{2 \times 15} \times \sqrt{15} \stackrel{R_1}{=} \sqrt{2} \times \sqrt{15} \times \sqrt{15}$$

$$\Leftrightarrow D = \sqrt{2} \times (\sqrt{15})^2 = \boxed{15\sqrt{2}}$$

$$E = 5^4 \times 5^3 \stackrel{P_1}{=} 5^{4+3} = \boxed{5^7}$$

$$F = \frac{5^3}{5^2} \stackrel{P_2}{=} 5^{3-2} = 5^1 = \boxed{5}$$

$$G = 2 + \frac{1}{4} \stackrel{F_0}{=} \frac{2 \times 4}{4} + \frac{1}{4} \stackrel{F_1}{=} \frac{8+1}{4} = \boxed{\frac{9}{4}}$$

$$H = \frac{8}{7} - \frac{9}{2} \stackrel{F_0}{=} \frac{8 \times 2}{7 \times 2} - \frac{9 \times 7}{2 \times 7} \stackrel{F_1'}{=} \frac{16-63}{14} = \boxed{-\frac{47}{14}}$$
