Chapitre 1

SIMPLIFICATION D'EXPRESSIONS

EXERCICES

Pour les exercices 1.1 à 1.6, simplifier au maximum l'expression donnée.

Exercice 1.1

$$A = \frac{\frac{3}{8}}{\frac{6}{12}}$$

$$B = \frac{3}{\frac{8}{\frac{6}{12}}}$$

$$C = \frac{\frac{3}{8}}{\frac{6}{12}}$$

$$D = \left(\sqrt{2} - 3\right)\left(\sqrt{2} + 3\right)$$

$$E = \sqrt{3^2 + 4^2}$$

$$F = \sqrt{3^2} + \sqrt{4^2}$$

$$G = \frac{3}{7} + \frac{3}{5}$$

$$H = \left(3 + \sqrt{2}\right)^2$$

$$I = \left(3\sqrt{2}\right)^2$$

$$J = -\frac{5}{18} - \frac{7}{6} + \frac{3}{9} - 2$$

$$K = 3 \times 3^n$$

$$L = \frac{9 \times 10^3}{1,8 \times 10^5}$$

$$M = 2\sqrt{3} - 7\sqrt{3} + 2\sqrt{27}$$

$$N = \frac{1}{\frac{1}{2} + \frac{1}{3}}$$

Exercice 1.2

$a = \dfrac{0,25}{2} \times 6$

$b = \dfrac{5 \times 4^n}{3 \times 2^n}$ où $n \in \mathbb{N}$

$c = \dfrac{0,8 \times 4^4 \times 9^3}{0,6^2 \times 12^5}$

$d = \left(\dfrac{2}{3}\right)^2 + 5 \times \dfrac{4}{3}$

$e = \sqrt{3^5 \times 15 \times 5^3}$

$f = \dfrac{\dfrac{14}{15}}{\dfrac{7}{12}}$

$g = \dfrac{15 \times 5^{n+1}}{15^{n+2}}$ où $n \in \mathbb{N}$

$h = \dfrac{7}{3} + \dfrac{5}{7}$

$i = \left(2 - \sqrt{3}\right)^2$

$j = \sqrt{2^3 \times 49} \times \sqrt{\dfrac{2^7 \times 9}{4900}}$

$k = \dfrac{8^2 \times 5^5 \times 7}{2^8 \times 5^4 \times 7^2}$

$l = \dfrac{\dfrac{3}{2} - \dfrac{5}{6}}{\dfrac{1}{4} + \dfrac{1}{3}}$

$m = \sqrt{98} + \sqrt{32} - \sqrt{8}$

$n = \dfrac{4 - \dfrac{2}{3}}{4}$

Exercice 1.3

Dans tout cet exercice, $n, p \in \mathbb{N}$.

$$A = \frac{7}{3} - \frac{\dfrac{8}{3}}{-\dfrac{5}{16}}$$

$$B = \frac{5 \times 10^3 \times 2}{0,25 \times 10^{-4}}$$

$$C = \frac{4 \times 2^{n+2}}{2^n}$$

$$D = \frac{12}{5}\left(\frac{\dfrac{1}{2} - \dfrac{1}{3}}{\dfrac{1}{3} - \dfrac{1}{4}}\right)\left(\frac{\dfrac{1}{7} - \dfrac{1}{5}}{\dfrac{5}{7} - \dfrac{7}{5}}\right)$$

$$E = \left(\sqrt{3} + \sqrt{5}\right)^2 - 2\sqrt{15}$$

$$F = 7^{n+1} - 3 \times 7^n$$

$$G = 3\left(2 - \frac{1}{3}\right)^2 - \frac{5}{2}$$

$$H = 2\pi - \frac{\pi - 7}{5}$$

$$I = \left(\frac{\dfrac{1}{2} - \dfrac{3}{4}}{\dfrac{1}{3} - \dfrac{4}{5}}\right)\left(\frac{\dfrac{4}{15} - \dfrac{1}{3}}{\dfrac{1}{6} + \dfrac{1}{4}}\right)$$

$$J = \frac{\left(\sqrt{3}\right)^{2p}}{3^p}$$

$$K = \frac{\dfrac{1}{3} - 3}{\left(\dfrac{2}{3} - 2\right)^2}$$

$$L = \frac{3}{2}\left(\sqrt{2} - \sqrt{3}\right)^2 + 3\sqrt{6}$$

$$M = 2^{2n} + 3 \times 4^n$$

$$N = \frac{\dfrac{4}{5} + \dfrac{5}{4}}{3 \times \dfrac{7}{2} - 2}$$

Exercice 1.4

Dans tout cet exercice, $n \in \mathbb{N}$.

$$A = 2 \times 10^7 \times 5 \times \frac{\left(10^{-3}\right)^5}{17+3}$$

$$B = 5 + \frac{\pi - 3}{7}$$

$$C = \frac{3\sqrt{28} - 4}{-2 + 3\sqrt{7}}$$

$$D = 5\left(4\sqrt{2}\right)^2$$

$$E = 6 \times 3^n + 2 \times 3^{n+2}$$

$$F = \frac{2^8 - 2^{10}}{2^8 + 2^{10}}$$

$$G = 5 + \left(4\sqrt{2}\right)^2$$

$$H = \frac{-21}{-\frac{3}{7} + \frac{3}{5}}$$

$$I = 2\sqrt{27} - \sqrt{15} \times 3\sqrt{45}$$

$$J = \frac{49}{24} \times \frac{25}{14} \times \frac{6}{5}$$

$$K = \left(\sqrt{3} - \sqrt{7}\right)^2 + \left(\sqrt{7} + \sqrt{3}\right)^2$$

$$L = \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} + \sqrt{3}}$$

$$M = \frac{2}{3} + \frac{3}{2} - \frac{1}{5}$$

$$N = \frac{\sqrt{3} \times 7\sqrt{12}}{2\sqrt{27}}$$

$$O = 3^n \left(\left(\frac{2}{3}\right)^n - \left(\frac{2}{3}\right)^{n+1}\right)$$

$$P = \frac{\frac{7}{5} - \frac{5}{7}}{4}$$

$$Q = \frac{10}{33} \times \frac{55}{4} \times \frac{1}{25}$$

Exercice 1.5

Dans tout cet exercice, $n \in \mathbb{N}$ et $a, b \in \mathbb{R}^*$.

$$A = -\frac{2n+3}{2^n} + \frac{4(n+1)+1}{2^{n+1}}$$

$$B = \frac{\frac{49}{24} \times \frac{25}{14} \times \frac{42}{5}}{\frac{5}{2}}$$

$$C = \frac{(9b^n)^2}{3b^2}$$

$$D = \frac{4 \times 2^{n+2}}{4^n}$$

$$E = \frac{2 - 4\sqrt{5}}{3 + 2\sqrt{5}}$$

$$F = \frac{\left(\left(1+\sqrt{5}\right)^2 - \left(1-\sqrt{5}\right)^2\right)^{2n}}{\left(\left(2\sqrt{5}\right)^2\right)^n}$$

$$G = 3\left(\sqrt{3}\right)^3 - 2\sqrt{75} + 5\sqrt{3^5}$$

$$H = \frac{7^{n+1} - 3 \times 7^n}{2 \times 7^n}$$

$$I = \frac{3\left(\frac{5}{7} - \frac{1}{3}\right)\left(\frac{1}{4} + \frac{5}{2}\right)}{3 - \frac{7}{6}}$$

$$J = \frac{a^{2n} + 2a^n}{4a^n}$$

$$K = 1 - \frac{\frac{2}{3}\left(1 - \frac{7}{2}\right)}{8 - \frac{5}{3}}$$

$$L = \frac{3 - \frac{2}{3} \times \frac{5-2}{8-2}}{1 - 2 \times \frac{9}{8}}$$

$$M = \left(\frac{2-\sqrt{3}}{\sqrt{3}-1}\right)^2$$

$$N = \frac{\left(3 - \frac{2}{3}\right) \times \frac{5-2}{8-2}}{(1-2) \times \frac{9}{8}}$$

Exercice 1.6

Dans tout cet exercice, $n, p \in \mathbb{N}$.

$$A = \left(1 - \sqrt{5}\right)^4 - \left(2\sqrt{5}\right)^3$$

$$B = 2 - \frac{2 \times \left(\frac{7}{4} - \frac{5}{3}\right)}{\left(\frac{1}{5} + \frac{3}{2}\right)\left(\frac{5}{2} - \frac{5}{3}\right)}$$

$$C = \sqrt{6} + \frac{\sqrt{2} + \sqrt{3}}{2\sqrt{2} - 5\sqrt{3}}$$

$$D = \left(4 - 3\sqrt{2}\right)^2 + 3\left(\sqrt{2} - 1\right)^2$$

$$E = \frac{\frac{1}{2} - \frac{1}{4} \times \left(3 + \frac{1}{3}\right)}{\left(\frac{1}{2} - \frac{1}{4}\right) \times 3 - \frac{1}{3}}$$

$$F = \frac{5 + 7\sqrt{2}}{\sqrt{2} - \sqrt{3}} + \frac{7 - 5\sqrt{3}}{\sqrt{2} + \sqrt{3}}$$

$$G = \left(\sqrt{2}\right)^{2n} + \frac{6^{n+1}}{5 \times 3^n}$$

$$H = \frac{\frac{1}{4} \times \left(2 - \frac{4}{5}\right) - \frac{5}{16}}{\frac{8}{15} \times 3 + \frac{6}{5}}$$

$$I = \frac{6\frac{5 \times \frac{3}{8} + \frac{7}{12} - 1}{7}}{-4 \times \frac{9}{15} + 3}$$

$$J = \frac{6 \times 9^n + 2 \times 3^{2n+2}}{6^3}$$

$$K = \frac{18^n + 2^n \times 3^{2n+2}}{6^{n-2}}$$

$$L = \frac{4}{7}\left(\sqrt{14}\right)^{2p+2}$$

$$M = \frac{10}{2\sqrt{5} - 5} - 2\frac{7 - 3\sqrt{5}}{\sqrt{20} + 5}$$

$$N = 4\left(2 - \frac{\sqrt{3}}{2}\right)^3$$

CORRECTION DES EXERCICES

Exercice 1.1

$$A = \frac{\frac{3}{8}}{\frac{6}{12}} \overset{\mathsf{F}_3}{=} \frac{3}{8} \times \frac{12}{6} \overset{\mathsf{F}_0}{=} \frac{3}{8} \times \frac{2}{1} \overset{\mathsf{F}_2}{=} \frac{3 \times 2}{8} = \frac{6}{8} \overset{\mathsf{F}_0}{=} \boxed{\frac{3}{4}}$$

$$B = \frac{\frac{3}{8}}{\frac{6}{12}} \overset{\mathsf{F}_3}{=} \frac{3}{\frac{8}{1} \times \frac{12}{6}} \overset{\mathsf{F}_0}{=} \frac{3}{\frac{8}{1} \times \frac{2}{1}} = \boxed{\frac{3}{16}}$$

$$C = \frac{\frac{\frac{3}{8}}{\frac{6}{12}}}{12} \overset{\mathsf{F}_3}{=} \frac{\frac{3}{1} \times \frac{6}{8}}{12} \overset{\mathsf{F}_0}{=} \frac{\frac{3}{1} \times \frac{3}{4}}{\frac{12}{1}} \overset{\mathsf{F}_2,\mathsf{F}_3}{=} \frac{9}{4} \times \frac{1}{12} \overset{\mathsf{F}_0}{=} \frac{3}{4} \times \frac{1}{4} = \boxed{\frac{3}{16}}$$

$$D = \left(\sqrt{2} - 3\right)\left(\sqrt{2} + 3\right) \overset{\mathsf{D}_4}{=} \left(\sqrt{2}\right)^2 - 3^2 = 2 - 9 = \boxed{-7}$$

$$E = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} \overset{\mathsf{R}_0}{=} \boxed{5}$$

$$F = \sqrt{3^2} + \sqrt{4^2} \overset{\mathsf{R}_0}{=} 3 + 4 = \boxed{7}$$

$$G = \frac{3}{7} + \frac{3}{5} \overset{\mathsf{F}_0}{=} \frac{3 \times 5}{7 \times 5} + \frac{3 \times 7}{5 \times 7} \overset{\mathsf{F}_1}{=} \frac{15 + 21}{35} = \boxed{\frac{36}{35}}$$

$$H = \left(3+\sqrt{2}\right)^2 \overset{\mathsf{D}_3}{=} 3^2 + 2\times 3\times\sqrt{2} + \left(\sqrt{2}\right)^2$$
$$\iff H = 9 + 6\sqrt{2} + 2 = \boxed{11+6\sqrt{2}}$$

$$I = \left(3\sqrt{2}\right)^2 \overset{\mathsf{P}_1'}{=} 3^2 \times \left(\sqrt{2}\right)^2 = 9\times 2 = \boxed{18}$$

$$J = -\frac{5}{18} - \frac{7}{6} + \frac{3}{9} - 2 \overset{\mathsf{F}_0}{=} -\frac{5}{18} - \frac{7\times 3}{6\times 3} + \frac{3\times 2}{9\times 2} - \frac{2\times 18}{18}$$
$$\iff J \overset{\mathsf{F}_1,\mathsf{F}_1'}{=} \frac{-5-21+6-36}{18} = \frac{-56}{18} \overset{\mathsf{F}_0}{=} \boxed{-\frac{28}{9}}$$

$$K = 3\times 3^n \overset{\mathsf{P}_1}{=} \boxed{3^{n+1}}$$

$$L = \frac{9\times 10^3}{1,8\times 10^5} \overset{\mathsf{P}_1}{=} \frac{9\times 10^3}{1,8\times 10^1\times 10^4} = \frac{9\times 10^3}{18\times 10^4} = \frac{9\times 10^3}{2\times 9\times 10^4}$$
$$\iff L \overset{\mathsf{F}_0,\mathsf{P}_2}{=} \frac{1}{2\times 10^{4-3}} = \frac{1}{2\times 10^1} = \boxed{\frac{1}{20}}$$

$$M = 2\sqrt{3} - 7\sqrt{3} + 2\sqrt{27}$$
$$\iff M = 2\sqrt{3} - 7\sqrt{3} + 2\sqrt{9\times 3}$$
$$\iff M \overset{\mathsf{R}_1}{=} -5\sqrt{3} + 2\sqrt{9}\sqrt{3}$$
$$\iff M \overset{\mathsf{R}_0}{=} -5\sqrt{3} + 2\times 3\sqrt{3}$$
$$\iff M = -5\sqrt{3} + 6\sqrt{3}$$
$$\iff M = \boxed{\sqrt{3}}$$

$$N = \frac{1}{\frac{1}{2}+\frac{1}{3}} \overset{\mathsf{F}_0}{=} \frac{1}{\frac{3}{2\times 3}+\frac{2}{3\times 2}} \overset{\mathsf{F}_1}{=} \frac{1}{\frac{3+2}{6}} \overset{\mathsf{F}_3}{=} \boxed{\frac{6}{5}}$$